Description of various parameters in **Altimeter LCS-Cores and Stretching Directions (Application to Oil-Spill Nowcasting)**

Lagrangian Coherent Structure Cores (LCS -Cores)

Lagrangian Coherent structures(LCS) arise in Ocean due to non-linear dynamics of Ocean. These 2-D structures have an ability to facilitate or block the material transport (of seawater + passive tracers) through them, thus organizing the flow pattern of passive tracers in the ocean. Its computation involves the 2-D advection of particles (starting from a well-defined grid) at a current time with altimeter velocities to a certain period of time (15 days in this case). Details can be found in Farazmand et al. 2012 and Onu et al. 2015. If the advection is carried out backward in time, then we can calculate the `attracting LCS`. These attracting LCS's form the flow pattern of the passive tracer. These LCS's can be termed as 'skeleton of flows' for a passive tracer. Passive tracer can be a Lagrangian float near surface, **Oil Spills, Chlorophyll**, occasionally **SST** and **Salinity**. **LCS-Cores** are the strongest of the LCS and these will last longer in time relative to other LCS's. LCS-Cores strongly affect the passive tracers over a larger period to time. Typical time-scale of these LCS-Cores can be few days. Details of computation of LCS-Cores can be found in Olascoaga et al. 2012. Strength of the LCS-Core is determined by the average lagrangian strain-rate (unitless quantity). These LCS Cores form fingering type instabilities along the **stretching directions** for an oil spill patch as illustrated in the below graphic. Other LCSs also affect the oil spill patch shapes but the LCS Cores are the most prominent among them.

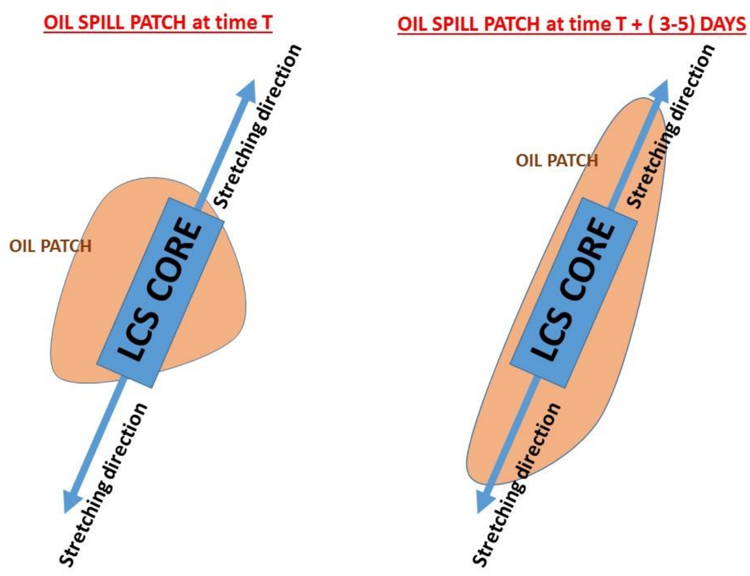

Figure 1. Illustration of stretching of an oil-patch if it happens to be on the LCS Core. This is a fingering type instability.

In the GUI, we have presented the **LCS-Cores**. These have been calculated form the altimeter velocity field. Duration of advection is 15 days.

LCS- Cores are shown as Scattered points in the plots. The color coding shows their strength (unit-less).

Stretching directions

As discussed in the previous section, the stretching directions are the directions associated with the LCS-Core. It is computed as the eigenvectors of the Cauchy-Green Tensor obtained through advection of particles. In physical sense, it means the direction of maximum relative stretching. Figure 1 shows the impact of stretching directions. We expect the passive tracers to align to these directions.

In the GUI, the arrow direction corresponds to the **stretching direction** of the LCS.

Background Field – FiniteTime Lyapunov Exponent (FTLE)

This scalar quantity is an intermediate field coming in calculation of LCS and LCS-Cores. This is a quantity that characterizes the rate of separation of infinitely close trajectories (of advected particles backward in time). Higher values signify relatively larger rate of separation than its neighboring locations. It is computed by advecting the particles for a particular time (15 days in this case) and finding the highest eigenvalue of the Cauchy green tensor that arise from advection. Details of computation can be found in Farazmand et al. 2012.

Ridges of FTLE forms generally contains LCS although it is not the sufficient condition. But it provides a nice picture of the overall structures present in the ocean arising from velocity field for a particular day.

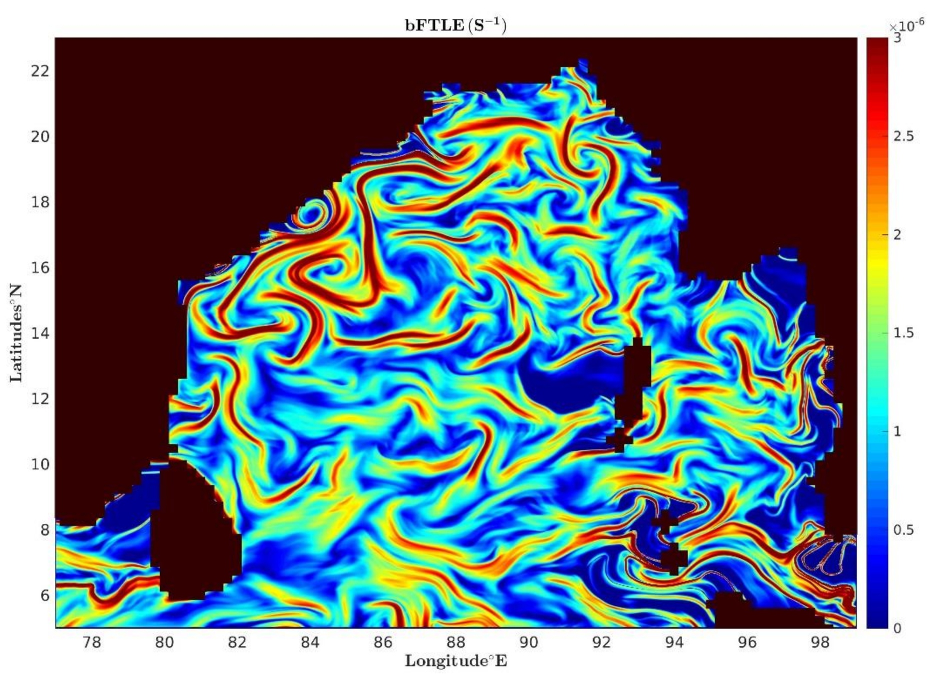

Figure 2. Backward time FTLE filed for a particular day (4th March 2016) in the Bay of Bengal calculated using the altimeter velocity field.

In the GUI, background is the FTLE field in (day $^{-1}$). Color coding is in greyscale.

Methodology

Differential equation for the two dimensional incompressible fluid motion is

$$\dot{\boldsymbol{x}} = \boldsymbol{v}(x,t)\,, \qquad \nabla.\boldsymbol{v} = 0 \qquad \textbf{(1)}$$

where the $\dot{x}$ represents the differential rate with respect to time and ∇ is the gradient with respect to variable x, and the velocity field $\boldsymbol{v}(x,t)$ denotes the time and location dependent velocity field (from altimeter) of the fluid.

For calculating the Lagrangian Coherent structures in two dimensions the following algorithm is given by Farazmand et al., 2012 and Onu et al. 2015.

In order to calculate the strength of LCSs we the $r(\boldsymbol{x}_t, t)$ parameter as given in Olascoaga et al. 2012.

The instantaneous normal attraction rate of an LCS at point $\boldsymbol{x}_t$ with unit normal $\boldsymbol{n}_t$ is measured by the Lagrangian strain rate

$$r(\boldsymbol{x}_t, t) = \langle \boldsymbol{n}_t, S(\boldsymbol{x}_t)\boldsymbol{n}_t \rangle \qquad \textbf{(2)}$$

where $\boldsymbol{S} = \frac{1}{2}[\nabla\boldsymbol{v} + (\nabla\boldsymbol{v})^*]$ is the rate-of-strain tensor.

If the value of $r(\boldsymbol{x}_t, t)$ is negative at all times, then we say that these point are LCS-Cores. The averaged value of $|r(\boldsymbol{x}_t, t)|$ is the measure of strength of the LCS-Core.

References

Olascoaga, María J., and George Haller. 2012. "Forecasting sudden changes in environmental pollution patterns." *Proceedings of the National Academy of Sciences* (National Acad Sciences). doi:10.1073/pnas.1118574109.

Onu, K., Florian Huhn, and George Haller. 2015. "LCS Tool: A computational platform for Lagrangian coherent structures." *Journal of Computational Science* (Elsevier) 7: 26-36. doi:10.1016/j.jocs.2014.12.0012.

Farazmand, Mohammad, and George Haller. 2012. "Computing Lagrangian coherent structures from their variational theory." *Chaos: An Interdisciplinary Journal of Nonlinear Science* (AIP) 22: 013128. doi:10.1063/1.3690153.